**On the Generalized Trapezoidal Inequality for Convex Function**

**Mehmet Zeki SARIKAYA**

*Department of Mathematics, Faculty of Science and Arts, Düzce University, Düzce-TURKEY*

sarikayamz@gmail.com

**Abstract.**  In the paper, we establish some new inequalities for differentiable convex functions, which are connected with Hermite-Hadamard-Fejer integral inequalities, and we present new generalized inequalities of trapezoidal type which cover the previously puplished results.

# IntroductIon

The inequalities discovered by C. Hermite and J. Hadamard for convex functions are considerable significant in the literature (see, e.g.,[3], [16], [24]). These inequalities state that if  is a convex function on the interval  of real numbers and  with , then

  (1.1)

where  is a convex function on the interval  of real numbers and  with  . A function  , is said to be convex if the following inequality holds



for all  and  We say that  is concave if  is convex.

The inequalities (1.1) have grown into a significant pillar for mathematical analysis and optimization, besides, by looking into a variety of settings, these inequalities are found to have a number of uses. What is more, for a specific choice of the function f, many inequalities with special means are obtainable. Hermite Hadamard's inequality (1.1), for example, is significant in its rich geometry and hence there are many studies on it to demonstrate its new proofs, refinements, extensions and generalizations. You can check ([1]-[18]) and the references included there.

In [4], Dragomir and Agarwal proved the following inequality connected with the right part of (1.1).

 **Theorem 1.** *Let  be a differentiable mapping on  ,  with  . If  is convex on  , then the following inequality holds:*

** (1.2)

In [6], Pearce and Pěcarić proved the following inequality.

**Theorem 2.** *Let  be a differentiable mapping on  ,  with  . If  is convex on  , for some  then we have*

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In this article, using functions whose derivatives absolute values are convex, we obtained new inequalities of Hermite-Hadamard-Fejer type. The results presented here would provide extensions of those given in earlier works.

# MaIn Results

In order to prove our main results, we need the following lemma:

 **Lemma 1.** *Let  be a differentiable function on  with  and let  be a continuous function. If  , then the following identity holds:*

 ** (2.1)

 *Proof.* By integration by parts,



we get the desired result (2.1).

 **Remark 1.** *If we take  and  in Lemma l1, we have*

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*which is proved by Dragomir and Agarwal in [4].*

**Theorem 4.** *With the assumptions of Lemma1. If  is a convex function on  , then the following inequality holds:*

**

**

*Proof.* By using Lemma 1, we have

 (2.2)

Since  is convex on  , we get

 (2.3)

and

 (2.4)

By substituting (2.3) and (2.4) in (2.2) we have





This completes the proof.

 **Remark 2.** *If we take  and  in Theorem 3, we have*

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*which is proved by Dragomir and Agarwal in [4] where we have used the fact*

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Another similar result is embodied in the following theorem

**Theorem 12**  *With the assumptions of Lemma l1. If   is a convex function on  , then the following inequality holds:*

** (2.5)

*where  and*

**

*Proof.* By using Lemma 1 and Hölder's integral inequality, we find that

 (2.6)

Since  is convex on  , we get

 (2.7)

and

 (2.8)

By substituting (2.6) and (2.7) in (2.5) we have



Thus, we get the desired result (2.5).

 Remark *If we take  and  in Theorem 4, we obtain the following first inequality*

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 *where we have used the fact*

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*For the proof of second inequality, let    and  Using the fact that*

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*and  then the desired result can be obtained straightforwardly.*

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